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# CALCULATION OF EFFICIENCY AND FREQUENCY CHARACTERISTICS OF CYCLOTRON GENERATORS OPERATING AT HARMONICS OF THE GYROFREQUENCY

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ABSTRACT. Based on the nonlinear theory, the amplitude and phase balance of a cyclotron and generator with a helical electron beam in a hollow or open resonator is analyzed. The efficiency of the generator at the first and second harmonics of the cyclotron frequency is investigated as a function of the ratio of working current to starting current and the detuning of the cyclotron frequency of the magnetic field from the natural frequency of the resonator. The results of calculations allow an estimation of the optimal generator parameters for provision of the maximum efficiency and the demands placed on its elements from the point of view of stability of the frequency.

In preliminary investigations of the power characteristics of generators with helical electron trajectories, the authors have analyzed problems of the effectiveness of the interaction of a broad beam with a transversely heterogeneous resonator field at the first, second and third harmonics of the cyclotron frequency, based on the balance of active power alone[2]. Actually, the active and reactive components of power are interrelated; the operating frequency, unknown in advance, must be determined along with the efficiency. /996[1]

In this work, based on the balance of the combined power, we solve the problem of the interaction between a thin, spiralized electron beam and a resonator field on the assumption of a single form of oscillations; optimal values of parameters and of their variations are sought, for which the efficiency is not reduced to less than one half its maximal value. The instability of the generator frequency and the steepness of the electron retuning of the generator are estimated.

The equations of movement for electrons drifting in the direction of the z axis in the righthanded beam in the field of the standing wave

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[1] Numbers in the margin indicate pagination in the foreign text.
[2] Results reported at the XXIII All-Union Scientific Session of Scientific-Technical Society for Radioengineering and Electrical Communications imeni A. S. Popov dedicated to Radio Day, Moscow, May 1966.

$$\vec{E} = \vec{1}_x E_0 \sin ky \sin \omega t \tag{1}$$

under the condition of approximate equality of the signal frequency $\omega$ to the initial relativistic cyclotron frequency $\Omega \approx \eta_0 B_0 (1 - \frac{1}{2} \beta_0)$ or a multiple of this latter ($\omega \approx n\Omega$; n = 1,2,...) can be written in the form

$$\left.\begin{aligned} &\frac{d\rho_i}{dT} = A\rho_i^{n-1} \cos \Theta_i \\ &\frac{d\Theta_i}{dT} = -nA\rho_i^{n-2} \sin \Theta_i + \Phi + \mu (\rho_i^2 - 1) \\ &\rho_i (0) = 1, \ \Theta_i (0) = \frac{2\pi}{16} i, \ i = 1, 2, ..., 16 \end{aligned}\right\} \tag{2}$$ /997

Equation (2) corresponds to the case when the axis of the beam is located at the node of the field with n multiple or at the antinode when n is odd. If this condition is not observed or if the transverse distribution function of the field differs from sinusoidal, so that

$$\vec{E} = \vec{1}_x E_0 \psi (\Gamma Y) \sin \omega t, \tag{1a}$$

where $\Gamma$ is the constant of dimensionality of the reverse length, in the expression for A we should introduce the correcting factor $|\psi^{(n-1)}(\Gamma Y)|(\Gamma/k)^{n-1}$, where Y is the coordinate of the axis;

$$\left.\begin{aligned} &A = \frac{n^n \beta_\perp^{n-2}}{2^n n!} \frac{\eta_0 E_0 L}{c^2 \beta_\parallel} |\psi^{(n-1)} (\Gamma Y)| \left(\frac{\Gamma}{k}\right)^{n-1} \\ &\mu = \frac{n\Omega\beta_\perp^2 L}{2c\beta_\parallel}; \ \Phi = (\omega - n\Omega) \frac{L}{c\beta_\parallel}; \ T = \frac{z}{L} \end{aligned}\right\} \tag{3}$$

are the parameters and independent variable of equation system (2); L is the length of the system; $\rho$ is the ratio of the angular velocity to its initial value $c\beta_\perp$; $\theta$ is the effective phase of the field force; i is an index enumerating the discrete charge bunches which modulate the beam (16- bunches per period were used in the calculations); $c\beta_l$ is the initial value of the longitudinal velocity.

Let us now turn to an analysis of the equations for determination of the active and reactive components of the power of the interaction. It is convenient in this calculation to normalize the power to the constant $I_0 U_\perp$, thus introducing the complex "transverse" efficiency $\eta$

$$\eta_a = \mathrm{Re}\{\eta\};\ \eta_r = \mathrm{Im}\{\eta\}. \tag{4}$$

For an individual electron

$$\eta_{ai} = 1 - \rho_i^2, \tag{5}$$

from which, using (2), we produce

$$\frac{d\eta_a}{dT} = -\frac{A}{8}\sum_{i=1}^{16} \rho_i^n \cos\Theta_i;\ \eta_a(0) = 0. \tag{6}$$

The slowly changing mutual phasing of the rotary velocity of the electron and the force which it receives from the field can be clearly represented using an instantaneous vector diagram in a complex plane, where $\theta_i$ is the "lead" /998 angle of the first vector over the second vector (the velocity vector is placed along the real axis). The complex power of the interaction is proportional to the product of the amplitude of the velocity vector times the complex-conjugate amplitude of the force vector. From this, we get an expression for the reactive component $\eta$:

$$\frac{d\eta_r}{dT} = \frac{A}{8}\sum_{i=1}^{16} \rho_i^n \sin\Theta_i;\ \eta_r(0) = 0. \tag{7}$$

As we know from the theory of resonant systems

$$\eta_r(L) = 2Q_{ld}\frac{\omega - \omega_0}{\omega_0}\eta_a(L);\ \gamma \equiv \frac{\eta_r}{\eta_a} = 2Q_{ld}\frac{\omega - \omega_0}{\omega_0}. \tag{8}$$

where $Q_{ld}$ is the loaded Q of the resonator; $\omega_0$ is its natural frequency. Relationship (8) allows us to determine the "hot" frequency of the generator $\omega$.

A stricter conclusion of equation (7) is based on analysis of the integral through the volume of the area of interaction from the scalar product of the current density times the intensity of the electrical field and subsequent transition to integration and averaging along the curved trajectory of the beam analogously to the method presented in [1, 2].

Let us now turn our attention to the fact that the value of $A^2$ [see (3)] is proportional to the energy stored in the resonator through which, in turn, the power output by the beam $\eta_a I_0 U_\perp$ is expressed linearly (with coefficient $\omega/Q_{ld}$). Due to this, the following relationship is correct:

$$\frac{I}{I_{st}} = \frac{\frac{A^2}{\eta_\perp}}{\lim\limits_{A\to 0}\left(\frac{A^2}{\eta_\perp}\right)}, \tag{9}$$

where $\eta_\perp \equiv \eta_a(L)$, $I_{st}$ is the minimum starting current of the device with the condition of optimization (regulation) of the value of the path angle $\Phi$ with values of all other parameters corresponding to the operating mode. The denominator of the right portion of (9) can be calculated by solving the system of equations (2), (6), (7) for sufficiently small A, or directly from the linear theory.

Using equation (9), as well as relationship (8), we can exclude from analysis the dimensionless amplitude A. As a result, the characteristics of oscillator ($\eta$,$\gamma$) can be expressed through the parameters $\Phi$, $I/I_{st}$ and the derivative $U_\perp N$ (volt-turns), proportional to parameter $\mu$. Parameter $\Phi$ can be be expressed as follows:

$$\Phi = \Phi_0 + \frac{\pi n N \gamma}{Q_{ld}}, \tag{10}$$

where

$$\Phi_0 = 2\pi n N\left(\frac{\omega_0}{n\Omega} - 1\right).$$

is the "cold" desynchronization parameter, which is dependent on the tuning of the resonator, the value of the magnetic field and the longitudinal velocity of the electrons, $N = L/n\lambda\beta_\parallel$ is the number of turns in the electron "spiral." In most cases, for high Q resonators, the supplementary term in (10) can be /999
ignored, and $\Phi$ can be replaced by $\Phi_0$.

The results of calculations are shown in Figures 1-4.

Figures 1a and 2a show the efficiency $\eta_\perp$ of generators at the first and second harmonics as functions of the desynchronization parameter $\Phi$ for $U_\perp N = 0.55\cdot 10^6 (n = 1)$ and $U_\perp N = 0.4\cdot 10^6 (n = 2)$ which, in addition to the optimal values of $I/I_{st}$, near 4, provides the maximum efficiency. The numbers on the curves show the values of $I/I_{st}$.

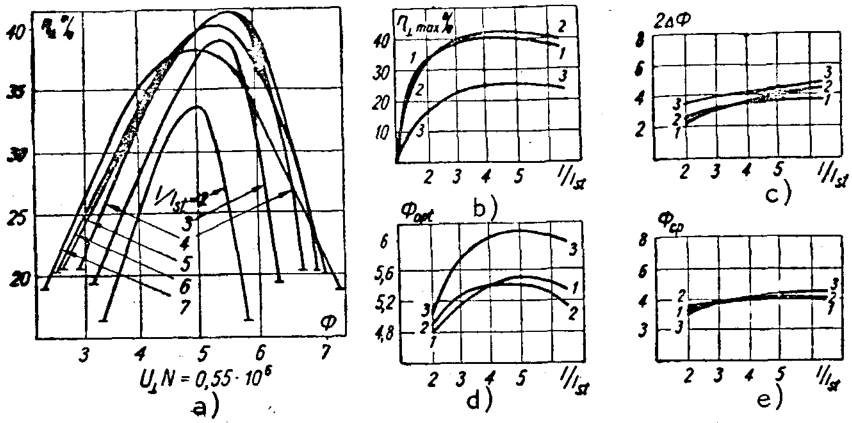


Figure 1. Main Characteristics of Cyclotron Generator at First Harmonic: a, Transverse efficiency as a function of path angle Φ for optimal volt-turns $U_{\perp}N$ with various $I/I_{st}$ ratios (ratios of working current to starting current); b, Efficiency with optimal Φ as a function of $I/I_{st}$; c, Optimal values of Φ; d, Interval 2ΔΦ, within which efficiency is reduced to one half; e, Mean value of Φ for range 2ΔΦ. $U_{\perp}N$ equal to:

1, $0.44 \cdot 10^6$; 2, $0.55 \cdot 10^6$; 3, $1.6 \cdot 10^6$

Figures 1b and 2b show the dependence of the efficiency, optimized with respect to Φ, on $I/I_{st}$ for three values of parameter $U_{\perp}N$, shown in the captions. We can see from the figure that a ratio of $I/I_{st}$ near 4 is optimal over a broad range of values of parameter $U_{\perp}N$.

The maximum efficiency values, corresponding to the optimal operating regimes, are 42% for the first harmonic and 32% for the second harmonic, which corresponds rather closely to the calculations of NIRFI [Scientific Research Institute for Radiophysics].

The dependence of the optimal values of Φ on $I/I_{st}$ is shown on Figures 1c and 2c. It is these values of Φ which were considered in constructing Figures 1b and 2b.

Figures 1d and 2d show the dependences of the width of the interval 2ΔΦ of the desynchronization parameter Φ, at the edges of which the efficiency $\eta_{\perp}$ is reduced to one half of its maximal value (for fixed $U_{\perp}N$), on $I/I_{st}$. These dependences allow us to estimate the permissible variations of the magnetic field and electron energies. For example, for an instrument optimized with respect to a combination of parameters (all of the numerical estimates encountered in the remainder of this paper will correspond to this example) $\Delta\Phi = 1.9$ where n = 1 and $\Delta\Phi = 1.6$ where n = 2. It follows from (3) and (10) that

$$\left.\begin{aligned} \left(\frac{\Delta B}{B}\right)_{max} &= \Delta\Phi \frac{\lambda}{L} \frac{\beta_{\parallel}}{2\pi} \text{ where } U = \text{const}, \; U_{\parallel} = \text{const} \\ \left(\frac{\Delta U}{U}\right)_{max} &\approx \Delta\Phi \frac{\lambda}{L} \frac{1}{\pi\beta_{\parallel}} \frac{U_{\parallel}}{U} \text{ where } B = \text{const} \end{aligned}\right\}. \quad (11)$$ /1000

The sign of equality in this latter relationship appears under the condition of fixed drift velocity of the electrons. Suppose $\beta_{\parallel} = 0.2$, $\frac{U}{U_{\parallel}} = 5$; then $\left(\frac{\Delta B}{B}\right)_{max} = 0.06\,\frac{\lambda}{L}$, $\left(\frac{\Delta U}{U}\right)_{max} \approx 0.6\frac{\lambda}{L}$ where n = 1 and $\left(\frac{\Delta B}{B}\right)_{max} = 0.05\frac{\lambda}{L}$, $\left(\frac{\Delta U}{U}\right)_{max} \approx 0.5\frac{\lambda}{L}$ where n = 2. Apparently, these same data can be used to estimate permissible heterogeneity of the magnetic field and dispersion of electron energies.

The average values of $\Phi$ for the intervals $2\Delta\Phi$ indicated above (generally speaking not corresponding with $\Phi_{opt}$) are presented on Figures 1d and 2d.

Figures 3a and 4a show the frequency as a function of desynchronization parameter $\Phi$, calculated for optimal volt-turns where n = 1, n = 2 for various $I/I_{st}$. These dependences can be used to calculate the steepness of retuning of the generator by a magnetic field or accelerating voltage. Over a broad range of values of path angle $\Phi$, the frequency changes approximately linearly. The dependence of $\Phi$ on magnetic field B is also linear, and under the condition

$$\frac{\lambda}{L} \frac{\Phi}{2\pi\beta_0} \sqrt{\frac{U_{\parallel}}{U}} \ll 1 \quad (12)$$

the dependence of $\Phi$ on U can also be considered linear [see (3)]. Therefore, in order to estimate the steepness of the electron retuning, we can use the relationships /1001

$$\left.\begin{aligned}\frac{\Delta\omega/\omega}{\Delta B/B} &= |g|\frac{L}{\lambda}\frac{2\pi}{\beta_{\parallel}Q_{ld}}\\ \frac{\Delta\omega/\omega}{\Delta U/U} &= -|g|\frac{L}{\lambda}\frac{\pi\beta_{\parallel}}{Q_{ld}}\frac{U}{U_{\parallel}}\end{aligned}\right\}. \qquad (13)$$

where $g = -|g|$ is the angular coefficient of the dependences $Q_{ld}(\omega - \omega_0)/\omega_0$ shown on the graphs of Figures 3a and 4a, on $\Phi$. For example, $|g| = 1.1$ where n = 1 and n = 2. Assuming $Q_{ld} = 500$, $\beta_{\parallel} = 0.2$, $U/U_{\parallel} = 5$, we have $(\Delta\omega/\omega)/(\Delta B/B) = 0.07(L/\lambda$ ("slow" adjustment by the magnetic field), $(\Delta\omega/\omega)/(\Delta U/U) = -0.007(L/\lambda$ ("rapid" adjustment by the intensity). These same figures characterize the relative stability of the frequency being generated with variations of current and voltage in the power supplies. In order to provide frequency stability $\Delta\omega/\omega = 10^{-4}$, it is necessary to maintain the solenoid current with an accuracy of 0.14 $\lambda/L$%, the voltage on the electrodes of the gun must be maintained with an accuracy of 1.4 $\lambda/L$%. Using Figures 3a and 4a, we can estimate also the effect of electron shift of the frequency (frequency as a function of current). When the current is increased to 4-5 $I_{st}$, the relative frequency shift $Q_s(\Delta\omega/\omega)$ is about 0.3 for n = 1 and 0.5 for n = 2 (with $\Phi = 4$). In addition to the factors mentioned, there is also frequency instability related to thermal expansion of the resonator. Apparently, the most easily achieved method of stabilizing the frequency is afc by voltage variation using frequency or phase discriminators. In particular, in order to compensate for instability of the magnetic field, the voltage must be adjusted according to the rule $\Delta U/U = (2/\beta_0^2)(\Delta B/B)$.

Figures 3b and 4b show the dependence of the width of the range of frequency adjustment on $I/I_{st}$. Here, a reduction of efficiency $\eta_{\perp}$ by a factor of 2 at the boundaries of the interval is assumed. Figures 3c and 4c show the displacement of the middle frequency of the adjustment range as a function of $I/I_{st}$. The graphs of Figures 3 and 4 were calculated for the same values of $U_{\perp}N$ as the graphs of Figures 1 and 2. The relative range of adjustment of the generator by frequency to produce a decrease in efficiency of a factor of 2 is expanded with increasing $I/I_{st}$ and with decreasing $U_{\perp}N$. For example, $2\Delta\omega/\omega = 1.5/Q_{ld}$ where n = 1 and n = 2, corresponding to 0.3% if $Q_{ld} = 500$. /1002

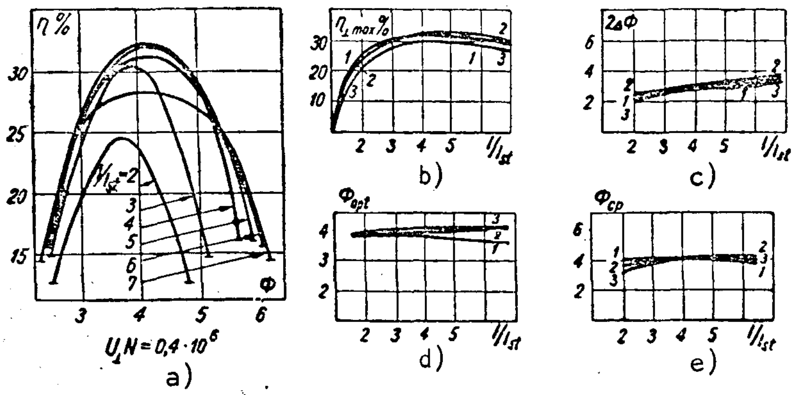


Figure 2. Principal Characteristics of Cyclotron Generator at Sceond Harmonic: a, Dependence of transverse efficiency on path angle Φ for optimal volt-turns $U_{\perp}N$ with various $I/I_{st}$ ratios (ratios of working current to starting current); b, Efficiency with optimal Φ as a function of $I/I_{st}$; c, Optimal values of Φ; d, Interval 2ΔΦ, within which efficiency is reduced to one half; e, Average value of Φ for indicated range 2ΔΦ. $U_{\perp}N$ equal to: 1, $0.33 \cdot 10^6$; 2, $0.4 \cdot 10^6$; 3, $0.51 \cdot 10^6$

In conclusion, we must note the following.

1. The large volume of the open resonator and the possibility of passing high beam currents through the system indicate that there is hope for successful utilization of cyclotron devices in various frequency ranges. The efficiency of cyclotron generators can be comparatively high.

2. The frequency characteristics of these devices are typical for devices using high Q resonator systems. The range of frequency adjustment is approximately 1.5 times as wide as the passband of the resonator. In order to achieve this adjustment, the accelerating voltage must be varied by 4-8%.

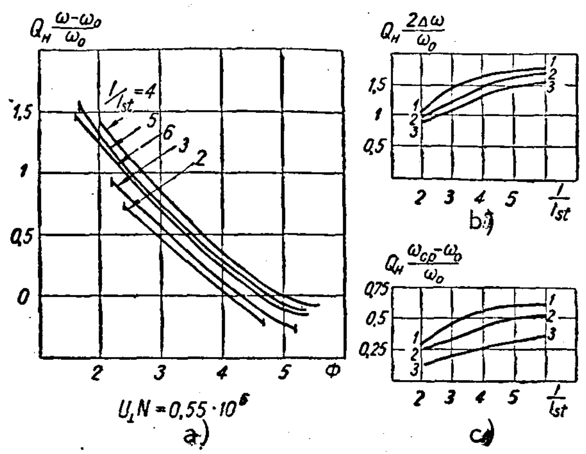


Figure 3. Frequency Characteristics of Cyclotron Generator at First Harmonic: a, Relative hot frequency as a function of path angle Φ for optimal volt-turns $U_\perp N$ with various values of ratio $I/I_{st}$ (ratio of working current to start current) for three values of volt-turns; b, Relative range of adjustment of frequency as a function of $I/I_{st}$; c, Relative value of middle frequency of range.
$U_\perp N$ equal to: 1, $0.44\cdot10^6$; 2, $0.55\cdot10^6$; 3, $1.6\cdot10^6$

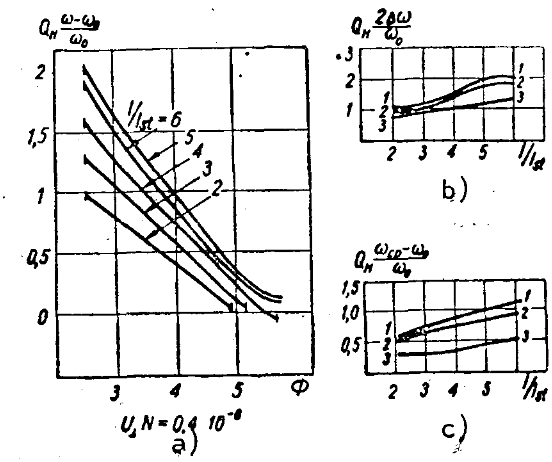


Figure 4. Frequency Characteristics of Cyclotron Generator at Second Harmonic Frequency: a, Dependence of relative hot frequency on path angle Φ for optimal volt-turns $U_\perp N$ with various values of ratio $I/I_{st}$ (ratio of working current to starting current); b, Dependence of relative range of adjustment of frequency on $I/I_{st}$; c, Relative value of middle frequency of range. $U_\perp N$ equal to: 1, $0.33 \cdot 10^6$; 2, $0.4 \cdot 10^6$; 3, $0.51 \cdot 10^6$

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